OLIMPIADA DE MATEMATICĂ

ETAPA LOCALĂ - 16 februarie 2014

Clasa a VI-a

VARIANTA 2

BAREM DE CORECTARE:

I. $P_n = 6^{1575} = 2^{1575} \cdot 3^{1575}$

Deducem $n = 2^x \cdot 3^y$; $x, y \in \mathbb{N}^*$ ....1p.

Divizorii lui *n* sunt: $1, 2, 2^2, \ldots, 2^x$

$3, 3 \cdot 2, 3 \cdot 2^2, \ldots, 3 \cdot 2^x$

$\vdots$

$3^y, 3^y \cdot 2, 3^y \cdot 2^2, \ldots, 3^y \cdot 2^x$ ....1p.

$P_n = \left(2 \cdot 2^2 \cdot 2^3 \cdot \ldots \cdot 2^x\right)^{y+1} \cdot \left(3 \cdot 3^2 \cdot 3^3 \cdot \ldots \cdot 3^y\right)^{x+1}$

$P_n = 2^{\frac{x(x+1)(y+1)}{2}} \cdot 3^{\frac{y(y+1)(x+1)}{2}}$ ....1p.

$P_n = 2^{1575} \cdot 3^{1575} \Rightarrow x(x+1)(y+1) = 3150$

şi $y(y+1)(x+1) = 3150$ ....1p.

$\frac{x}{y} = 1 \Rightarrow x = y$ ....1p.

$\Rightarrow x(x+1)^2 = 3150 = 14 \cdot 15^2 \Rightarrow x = y = 14$ ....1p.

$\Rightarrow n = 2^{14} \cdot 3^{14} = 6^{14}$ ....1p.

II. $\overline{abcd} = 5k + 2 \Rightarrow d \in \{2, 7\}$ ....1p.

$36 / \overline{abcd} \Rightarrow \overline{abcd}$ este număr par $\Rightarrow d = 2$ ....1p.

Cum $a - d = 4 \Rightarrow a = 6$ ....1p.

$4 \cdot 9 = 36$; $(4, 9) = 1 \Rightarrow 4 / \overline{6bc2}$ şi $9 / \overline{6bc2}$ ....1p.

$4/\overline{6bc2} \Rightarrow c \in \{1,3,5,7,9\}$ .... 1p.

$9/\overline{6bc2} \Rightarrow 9/6+b+c+2=b+c+8$ .... 1p.

Finalizare: Numerele $\overline{abcd}$ ce îndeplinesc condiţia problemei sunt:

6012; 6732; 6552; 6372; 6192 .... 1p.

III. $\overline{ab}=(a-b)(\overline{ba}-3)$

Notăm $a-b=x$; $x\in\mathbb{N}^* \Rightarrow a=x+b$ .... 1p.

Avem: $10a+b=x\cdot(10b+a-3)\Rightarrow 10(x+b)+b=$

$=x(10b+x+b-3)\Rightarrow 10x+11b=10xb+x^2+xb-3x$ .... 1p.

$\Rightarrow 13x-x^2=11xb-11b\Rightarrow x(13-x)=11b(x-1)$ .... 1p.

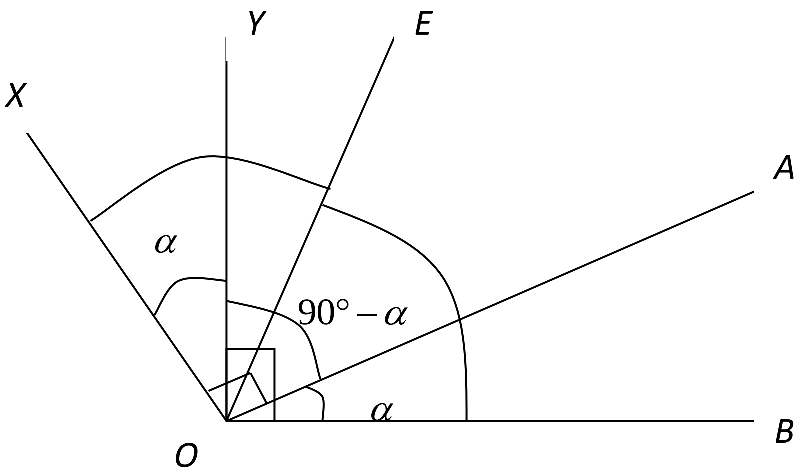
$\Rightarrow 11/x(13-x)$ .... 1p.

$\Rightarrow 11/x$ sau $11/13-x\Rightarrow x=2$ .... 1p.

$\Rightarrow 2\cdot 11=11\cdot b\Rightarrow b=2$ .... 1p.

$\Rightarrow a=2+2=4\Rightarrow \overline{ab}=42$ .... 1p.

IV.



a) $m(\sphericalangle AOB)=\alpha \Rightarrow m(\sphericalangle YOA)=90° \Rightarrow m(\sphericalangle XOY)=\alpha$ ....1p.

$m(\sphericalangle XOB)=90°+\alpha$

$(OE$ bisectoarea $\sphericalangle XOB \Rightarrow m(\sphericalangle XOE)=m(\sphericalangle BOE)=\frac{90°+\alpha}{2}$ ....1p.

Arătăm că $m(\sphericalangle YOB)>m(\sphericalangle BOE)>m(\sphericalangle AOB) \Leftrightarrow 90°>\frac{90°+\alpha}{2}>\alpha$ 1p.

i) $90°>\frac{90°+\alpha}{2} \Leftrightarrow 90°>\alpha$

ii) $\frac{90°+\alpha}{2}>\alpha \Leftrightarrow 90°>\alpha$ ....1p.

$\Rightarrow (OE \in Int(\sphericalangle AOY)$

b) $m(\sphericalangle XOE)=\frac{11}{4}\cdot m(\sphericalangle AOB) \Leftrightarrow \frac{90°+\alpha}{2}=\frac{11\alpha}{4}$ ....1p.

$\Leftrightarrow 180°+2\alpha=11\alpha \Leftrightarrow 9\alpha=180° \Leftrightarrow \alpha=20°$ ....1p.

$m(\sphericalangle AOB)=20°$; $m(\sphericalangle XOB)=20°+90°=110°$ ....1p.